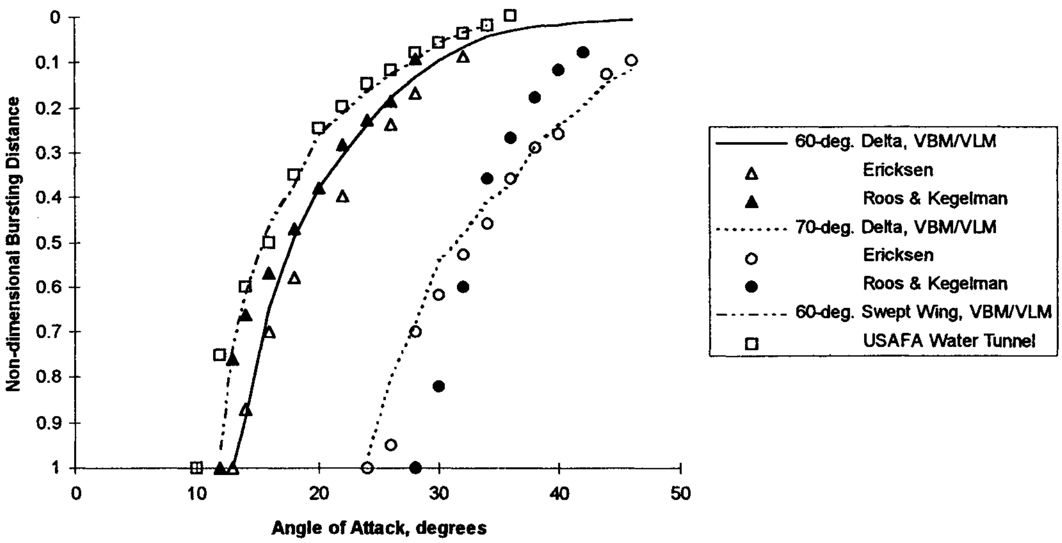

Fig. 2 Vortex burst trajectories.

The control parameter K_f in Eq. (9) is the only variable needed to fine-tune the vortex-burst predictions. It was adjusted for the 60-deg delta wing to achieve onset of vortex burst at the wing trailing edge at $\alpha = 13$ deg. A value of 2.6 for K_f gave this result. When this same value of K_f was used for VBM/VLM computations over a range of angles of attack for each of the wing shapes tested, the vortex burst locations shown in Fig. 1 resulted. Wind-tunnel values for burst locations on similar models from Refs. 8 and 9 are plotted in Fig. 2 for comparison. Note the good agreement between the model and wind-tunnel data for all shapes tested.

Conclusions

A model for leading-edge vortex bursting has been developed and used with the vortex lattice method to predict vortex bursting on highly swept wings. The model was derived from the steady, incompressible Navier–Stokes equations for the vortex core. The method was tested on four highly swept wing models. In all cases vortex burst predictions agreed well with the vortex burst locations observed in wind- and water-tunnel tests.

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Kill Probability in Antiaircraft Firing Theory

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Nomenclature

$P_r(x_1, x_2, x_3)$ = damage function[1] (herein after referred to as DF), which determines the value of probability of killing the target on the condition that the center of inertia of the target is in a defined location in space (x_1, x_2, x_3) related to the projectile blast epicenter.

P_r = the probability of killing the target when at least a single hit is scored. It is the average value of DF for the points (x_1, x_2, x_3) contained within the target.

$\boldsymbol{R}_b$ = random vector process or random vector variable that describes ballistic dispersion. It characterizes the location of a projectile relatively to the average trajectory.

$\boldsymbol{R}_t$ = random vector process or random vector variable that describes aiming error. This vector is characterized by a minimum length during the projectile flight and is equal to the difference between the radius vector $\boldsymbol{x}_p$ of the point that overlaps the

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c.m. of the projectile and the radius vector x_c of the hypothetical target.

R'_t = random vector process or random vector variable that describes corrected aiming error. This vector is characterized by a minimum length during the projectile flight and is equal to the difference between the radius vector x_p of the point that overlaps the c.m. of the projectile and the radius vector x_c of the actual target.

Introduction

THIS Note will show the kill probability of a small-caliber automatic AA cannon system against a single airborne target. A probability model investigates the change in probability value as a function of ballistic and meteorological conditions of shooting, number of rounds fired, technical characteristics of the armament, fire control system, target, etc. This model is therefore a tool for assessment of the existing AA systems as well as the ones being designed. It can also be used for the construction of models used for simulation of combat action. The theory has been illustrated with examples of numerical results.

The approach to probabilistic modeling of killing an airborne target may vary and depends on the number of data available and the objective function of such analysis.

Usually during the analysis of this problem the following two partial tasks are considered: 1) determination of DF, $P_r(x_1, x_2, x_3)$ and 2) determination of the probability of hitting the target or hitting the certain three-dimensional range containing the target, p.

DF characterizes each target–projectile configuration. These functions are determined on the basis of empirical data. Some examples of such functions can be found in a report of the Bofors Company.[2] In comparative analysis, instead of the DF, it is often accepted to use the probability of killing the target when at least a single hit is scored, P_r. The value of this probability is usually given as a function of projectile caliber[3] or mass of explosives in the projectile.[4]

The second of the two partial tasks is directly connected with the problem of firing at the airborne target. Figure 1 presents the scheme of such firing. In the moment t_0, the projectile is at the muzzle. The target is then at a point A'_s. However, target tracking system locates the target at a point A_s. It is the result of a target tracking error characterized by a vector $\boldsymbol{R}_s$. On the basis of the coordinates of A_s, the assumed hypothesis for the target motion during the flight of the projectile, ballistic, and meteorological conditions of shooting, the fire control system solves the hit problem. The solution gives the hypothetical location of the hit point A_w in the moment t_1. The existence of the aiming error $\boldsymbol{R}_t$ is why in the moment t_1 the projectile on average trajectory will be at a point P. Because of the ballistic dispersion $\boldsymbol{R}_b$ the actual location of the projectile in the moment t_1 is at point P'. Additionally, the actual location of the target in the moment t_1 is at point A'_w. The reason for that is the existence of the corrected aiming error $\boldsymbol{R}'_t$. Therefore, in the range of projectile and target rendezvous their relative location will be described by a vector $\boldsymbol{R}$. The latter is the vector sum of $\boldsymbol{R}'_t$ and $\boldsymbol{R}_b$. It has to be noted here that the optimization of the AA firing problem means the minimization of $\boldsymbol{R}$ in the range of projectile and target rendezvous. Because of the random character of this phenomenon, $\boldsymbol{R}$ should be treated as a random vector process or random vector variable. The determination of the probability of hitting the target or hitting the certain three-dimensional range containing the target p will be possible when the probability density function of $\boldsymbol{R}$ is known.

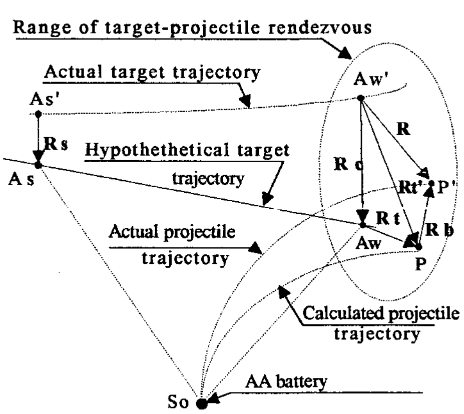

Fig. 1 AA firing scheme.

Additionally, the general model describing the kill probability should also take into consideration the number of rounds fired and type of fire, i.e., single cannon or battery, etc.

Characteristics of R_t, R'_t and R_b

In practical applications, especially in comparative analysis, it is often assumed that the errors $\boldsymbol{R}_t$ or $\boldsymbol{R}'_t$ are characterized by a three-dimensional vector normal variable with independent components, which probability density in the reference system associated with the target's center of inertia is expressed with the following formula:

$$f_t(x'_1, x'_2, x'_3) = \frac{1}{(2\pi)^{3/2}\sigma_{t1}\sigma_{t2}\sigma_{t3}} \exp\left\{-\frac{1}{2}\left[\frac{(x'_1 - w_1)^2}{\sigma_{t1}^2} + \frac{(x'_2 - w_2)^2}{\sigma_{t2}^2} + \frac{(x'_3 - w_3)^2}{\sigma_{t3}^2}\right]\right\} \tag{1}$$

where w_1, w_2, and w_3 denote mean values, and σ_{t1}, σ_{t2}, and σ_{t3} denotes standard deviations for each component.

Then the mean values w_1, w_2, and w_3 are the components of vector $\boldsymbol{w}$, which characterizes the systematic error $\boldsymbol{R}_t$ or $\boldsymbol{R}'_t$ and the standard deviations σ_{t1}, σ_{t2}, and σ_{t3} are the components of vector $\boldsymbol{\sigma}$, which characterizes the random error $\boldsymbol{R}_t$ or $\boldsymbol{R}'_t$. As shown on Fig. 1 the difference between $\boldsymbol{R}_t$ and $\boldsymbol{R}'_t$ equals $\boldsymbol{R}_c$. For the target velocity of 250 m/s flying at a distance of 3000 m from the AA battery firing position the absolute value of $\boldsymbol{R}_c$ can reach about 210 m for a 35 mm × 228 cartridge and even about 290 m for a 30 mm × 170 cartridge.

For the ballistic dispersion $\boldsymbol{R}_b$ the following is usually assumed in the range of projectile and target rendezvous:

1) The position of projectile in relation to the average trajectory is described by a normal random vector variable having independent components; the standard deviations of the components lying in a plane perpendicular to a tangent to the trajectory are equal ($\sigma_1 = \sigma_2 = \sigma$).

2) Projectile velocity is described by a determined vector whose components are equal to the average components of an adequate random process.

Then the probability density of the ballistic dispersion $\boldsymbol{R}_b$ is described in the reference system associated with the center of dispersion with the following formula:

$$f(x_1, x_2, x_3) = \frac{1}{(2\pi)^{3/2}\sigma^2\sigma_3} \exp\left\{-\frac{1}{2}\left[\frac{(x_1 + x_2)^2}{\sigma^2} + \frac{x_3^2}{\sigma_3^2}\right]\right\} \tag{2}$$

Additionally, it is assumed that the standard deviations are the functions of firing distance D as follows:

$$\sigma(D) = A \cdot D, \qquad \sigma_3(D) = B \cdot D, \tag{3}$$

where σ_3 is the standard deviation of the x_3 component oriented along the tangent to the trajectory.

According to Refs. 5 and 6 for the factors A and B one can assume the following values: $A = 0.0015$ (1/m) and $B = 0.00325$ (1/m). They are the mean values for new AA cannons in the range of calibers between 20–40 mm.

Probability of Killing with Kinetic Energy Projectile, Taking into Account the Motion of the Target

The discussed probabilistic model is a result of generalization of the papers by Pogorzelski[7] and Brändli.[4] In this analysis simplified models have been adopted to characterize $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$. These models were discussed in the previous paragraph. Substituting $\boldsymbol{R}_t$ by $\boldsymbol{R}_t'$ does not affect the procedure and the final formulas. The following two scenarios were the subject of analysis:

1) The first scenario is characterized by the lack of correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$, this practically means a situation in which the process of laying the gun is corrected for each shot in a burst.

2) The second scenario applies to the case where there exists a correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$. Attention was drawn to an extreme case when the whole burst was shot with the same $\boldsymbol{R}_t$.

Bearing in mind the possibility of experimental verification of the model it was assumed that the target is a rectangular shape moving with a velocity of v_c in a direction given by the longer side of the rectangular (Fig. 2). A right-handed Cartesian coordinate system $O'x_1'x_2'x_3'$ was associated with the target. A right-handed Cartesian coordinate system $Ox_1x_2x_3$ was associated with the center of ballistic dispersion. The system $Ox_1x_2x_3$ was moving with an average projectile velocity v_p. It was also assumed that the values of probability density of $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$ remain unchanged while firing a burst of N rounds. Generally, the coordinate systems $O'x_1'x_2'x_3'$ and $Ox_1x_2x_3$ are in relation to each other, translated and rotated.

Because of the complexity of the derivation of formulas that describe the probability of killing the target, the rest of the reasoning will be presented only in draft.

To determine the probability p of hit with a single shot, on the condition that $\boldsymbol{R}_t$ has the coordinates (m, n, and s), it was necessary to find in the plane of the target the location of a hit point (x_{p1}' and x_{p2}') by a projectile located in point P'. The relations between the coordinates in both systems $O'x_1'x_2'x_3'$ and $Ox_1x_2x_3$ are used to achieve this. These relations are in accordance with the theory of the affine transformation for a three-dimensional space. From the resulting formulas it is possible then to find the coordinates x_1 and x_2 as functions of x_{p1}', x_{p2}', x_3, v_{p3}, and v_c. Single-shot hit probability p can be obtained after the integration of formula (2) within the boundaries of the rectangular. The coordinates x_1 and x_2 have to be expressed as functions of x_{p1}', x_{p2}', x_3, v_{p3}, and v_c. This leads to the following formula:

$$p = \int_{-l}^{+l}\int_{-h}^{+h}\int_{-\infty}^{+\infty} f(x_{p1}', x_{p2}', x_3, v_{p3}, v_c, m, n, s)\, dx_{p1}'\, dx_{p2}'\, dx_3 \tag{4}$$

where $2h$ is the length of the side AB of the rectangular ABCD in Fig. 2 and $2l$ is the length of the side BC of the rectangular ABCD on Fig. 2.

The subsequent discussion applies to the second case when there exists a correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$. Then the whole burst was shot with the same $\boldsymbol{R}_t$ described by the coordinates (m, n, and s). Two alternatives were considered in which an event of target killing was characterized by a probability P_r or by a minimum number of required hits N_T. In both cases the reasoning is based on the Bernoulli's scheme of experiments.[8] In the first case the probability of success in an experiment equals the product pP_r. Then, according to the binominal distribution, the probability of at least one success in N experiments equals

$$P_R' = 1 - (1 - pP_r)^N \tag{5}$$

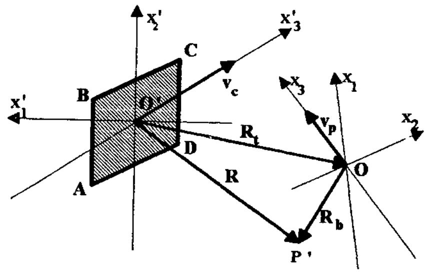

Fig. 2 Example location of the center of ballistic dispersion in relation to the target.

The probability of zero hits in a burst of N shots equals

$$P_M' = (1 - p)^N \tag{6}$$

and the probability of hitting, but not killing, the target in a burst of N shots equals

$$P_E' = 1 - P_R' - P_M' \tag{7}$$

In the second case the probability of hitting the target at least N_T times in a burst of N shots equals

$$P_{R1}' = 1 - \sum_{k=0}^{N_T-1} \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k} \tag{8}$$

The probability of missing the target in a burst of N shots equals

$$P_{E1}' = 1 - P_{R1}' - P_M' \tag{9}$$

The probabilities given in formulas (5–9) are the conditional ones. They relate to the case when $\boldsymbol{R}_t$ is described by the coordinates (m, n, and s). To calculate the relevant total probabilities, one has to use Bayes' formula.[8] This leads to the calculation of the desired probabilities:

1) Target killing P_R—when the event of target killing is taken into account according to the first alternative.

2) Target killing P_{R1}—when the event of target killing is taken into account according to the second alternative.

3) Hitting but not killing P_E—when the event of target killing is taken into account according to the first alternative.

4) Hitting but not killing P_{E1}—when the event of target killing is taken into account according to the second alternative.

5) Not hitting the target at all P_M—which probability is equal for both alternatives of considering an event of target killing

$$P_R = 1 - \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (1 - p\cdot P_r)^N f_t(m, n, s)\, dm\, dn\, ds \tag{10}$$

$$P_M = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (1 - p)^N f_t(m, n, s)\, dm\, dn\, ds \tag{11}$$

$$P_E = 1 - P_R - P_M \tag{12}$$

$$P_{R1} = 1 - \sum_{k=0}^{N_T-1} \frac{N!}{k!(N-k)!} \times \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p^k(1-p)^{N-k} f_t(m, n, s)\, dm\, dn\, ds \tag{13}$$

$$P_{E1} = 1 - P_{R1} - P_M \tag{14}$$

In these formulas p should be substituted according to formula (4) and the form of the function $f_T(m, n, s)$ is according to formula (1).

For the first scenario, which is characterized by the lack of correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$, calculations of each probability were done using the fact that the probability q of a miss in a single shot is given by formula (11) when one substitutes $N = 1$. Then the probability of hit with a single shot equals

$$p = 1 - q \tag{15}$$

and the other values of probabilities are defined by the formulas of the same structure as the formulas (5–9).

Probability of Killing by the Fragments of the Shell, Taking into Account the Motion of the Target

The presented probability models allow the fragmentation action of a round equipped with a proximity, time, or programmed fuse to be taken into account. It has to be noted that there is a difference between firing a round equipped with proximity fuse and a round with time or programmed fuse. In the first case one can assume that the random vector process describing the location of the round's blast epicenter in relation to the target is equal to $\boldsymbol{R}_t$ or $\boldsymbol{R}_t'$. In the second case the discussed random vector process is a function of $\boldsymbol{R}_t$ or $\boldsymbol{R}_t'$ and a random variable T describing the time from the moment the round leaves the barrel to the moment of blast. This variable takes into account the systematic error that originates in the dispersion of delayed burning time or the dispersion of a timer-counting mechanism.

In both cases of the presented models one should then substitute the conditional killing probability P_r by the DF $P_r(x_1, x_2, x_3)$. The form of formulas describing the probabilities P_R, P_{R1}, P_E, and P_{E1} would depend on the form of the DF, and consequently, each case should be analyzed individually.

Example Numerical Results

The calculations were made for an extreme case when a target of 8 m length by 2 m height moves at a distance of 3000 m, just above the ground level in a direction perpendicular to the axis of the barrel, with the velocity of 250 or 50 m/s. The AA battery consists of a twin 30 mm × 170 Mauser cannon ($N = 70$, $P_r = 0.37$) or a twin 35 mm × 228 Oerlikon cannon ($N = 35$, $P_r = 0.5$). The values of N and P_r were set according to Refs. 3 and 6. It was additionally assumed that for both calibers $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$ are the same and the respective standard deviations equal: $\sigma_{t1} = \sigma_{t2} = 1.8$ m; $\sigma_{t3} = 2.0$ m; $\sigma_1 = \sigma_2 = 4.5$ m, $\sigma_3 = 9.0$ m. The results are given in Tables 1 and 2.

Table 1 Values of probability when there is correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$

Probabilities	30 mm × 170 v_c = 250 m/s	35 mm × 228 v_c = 250 m/s	35 mm × 228 v_c = 50 m/s
P_R	0.3666	0.4688	0.4846
P_M	0.0091	0.0624	0.0309
P_E	0.6243	0.4688	0.4846

Table 2 Values of probability when there is no correlation between $\boldsymbol{R}_b$ and $\boldsymbol{R}_t$

Probabilities	30 mm × 170 v_c = 250 m/s	35 mm × 228 v_c = 250 m/s	35 mm × 228 v_c = 50 m/s
P_R	0.8314	0.7514	0.8279
P_M	0.0073	0.0584	0.0271
P_E	0.1613	0.1902	0.1415

Conclusions

The numerical results presented indicate the strong influence of the method of laying the gun on the final probability value. Correcting the aiming for each shot in a burst approximately doubles the probability P_R. However, it does not affect significantly the value of the probability P_M. It also has to be noted that there is relatively weak influence of the target velocity on the values of probability.

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Simulation of DD-963 Ship Airwake by Navier–Stokes Method

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Introduction

THE ship airwake is defined as an arbitrary volume of air, namely an air burble, surrounding the ship. The effect of airwake on aircraft/ship interface operation is determined by the airflow disturbances caused by the ship that are perceptible to the pilot during the final approach and landing in a shipboard operation. A better understanding of the airwake effect would improve aircraft operations in a seabase interface environment.[1,2]

The complexity of the ship airwake problem requires the use of a Navier–Stokes-type method to reveal correct flow features. Advances in numerical algorithm schemes along with increased computer speed and capacity have made it feasible

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